

Solution Sheet on Problem Set 4

**Fixed Income**

Deadline: 16.12.2021

**Solved by: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

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| **Task** |  | **Points Earned** |
| **Forwards to Interest**  a)  Different types of interest rates  (9 points) |  |  |
| b)  Z-Bond prices  (7 points) |  |  |
| c)  Forward Rate (3y to 20y) (4 points) |  |  |
| **Fix.Inc. Calculus**  a)  Coupon bond price (4 points) |  |  |
| b) Yield-to-maturity  (4 points) |  |  |
| c) Bond Duration’s  (6 points) |  |  |
| d) Scenarios: description (6 points) | **Adjusted Yield Curves:**    From the graphs above, we can observe that adding a constant to the yield function (+/-0.005) leads to a parallel upward/downward shift of the yield curve, which is in accordance with the underlying mathematical and geometric properties of a function. Additionally, multiplying the yield by a constant >/< 1 leads to a squeeze/stretch of the yield curve, as stated by the yellow graph (\*1.3). Hence, (almost) all yields across the maturities are higher, whereby the tails are converging to the original yields and the yield at 15-year maturity has the highest difference to the original yield. Lastly, the transformation by stretching combined with a dynamic/dependent upward shift based on the maturity (\*0.3+(maturity/4000)) leads to a transformation that has (strictly) increasing yields over the maturities. |  |
| e) Scenarios: YTM (10 points) | |  |  |  | | --- | --- | --- | |  | **Price** | **YTM** | | **Spot Rate** (Y’=Y+0.005) | 98.00151 | 0.010606 | | **Spot Rate** (Y’=Y-0.005) | 137.725946 | 0.000478 | | **Spot Rate** (Y’=Y\*1.3) | 109.886897 | 0.007174 | | **Spot Rate** (Y’=Y\*0.3+tau/4000) | 96.258536 | 0.011144 | |  |  |  | |  |
| f) Scenarios: prices (10 points) | **Prices vs. Approximated Prices:**    From the graph above, we can conclude that the price approximation using the dollar duration and the absolute change in the yield-to-maturity (YTM) leads to better results in the increasing YTM. For lower YTMs (close to zero), the approximation shows quite a large difference to the real price, whereas the approximation with a YTM >0.01 leads to very small approximation differences. |  |
| **3.**  **Yield Curve** a) Yield curve patterns (4 points) | **Yield Curve:**    From the yield curve, we can observe that the interest rates on government bonds for the maturities of one and three years break the theoretical concave structure. For this to hold, we would assume higher interest rates for these maturities and hence absolute increasing but marginally decreasing interest rates. Furthermore, due to the inflation-targeting low interest rate policy, i.e. in Europe and in the US, the yield curve is fully defined at negative interest rates for maturities up to 20 years. Lastly, for the short maturities of 1m, 2m and 3m, we would also assume slightly increasing interest rates, whereby equal rates are observed in the data. |  |
| b)  Z-bond prices (6 points) | **Zero Coupon Bond Term Structure:** |  |
| c) Yield curve fit (8 points) |  |  |
| d) 7 year z-bond (5 points) |  |  |
| e) Semi-annual coupon bond (5 points) |  |  |
| f) Level, slope & curvature (6 points) |  |  |
| g) Different Spreads (6 points) |  |  |